☐ Worksheet 3.1.1 Knowledge preview—straight-line motion

VECTORS

☐ Worksheet 3.1.7 Literacy review—talking motion
☐ Practical activity 3.1.1 Projectile motion—an introduction

PROJECTILE MOTION

☐ Worksheet 3.1.2 Projecting that missile—safe or not?
☐ Worksheet 3.1.3 Human cannonballs—a data analysis of projectile motion
☐ Worksheet 3.1.7 Literacy review—talking motion
☐ Practical activity 3.1.1 Projectile motion—an introduction
☐ **Mandatory practical 1 Projectile motion—the effect of launch angle on range**

INCLINED PLANES

☐ Worksheet 3.1.7 Literacy review—talking motion
☐ Practical activity 3.1.2 Forces down an incline

CIRCULAR MOTION

☐ Worksheet 3.1.4 Dashing around the corner
☐ Worksheet 3.1.5 Circular motion and gravity
☐ Worksheet 3.1.7 Literacy review—talking motion
☐ Practical activity 3.1.3 Circular motion—centripetal force in a horizontal plane
☐ Practical activity 3.1.4 Circular motion—centripetal force in a vertical plane

GRAVITATIONAL FORCE AND FIELDS

☐ Worksheet 3.1.6 Working with orbits
☐ Worksheet 3.1.7 Literacy review—talking motion

ORBITS

☐ Worksheet 3.1.6 Working with orbits
☐ Worksheet 3.1.7 Literacy review—talking motion

☐ Worksheet 3.1.7 Literacy review—talking motion
☐ Worksheet 3.1.8 Thinking about my learning
☐ **Topic Review 3.1**

Key knowledge

Vectors

Analysing motion in one dimension requires only simple mathematics. However, vectors in two dimensions may involve a change in direction, such as walking up a hill or running around a block. In these cases, more complex methods are needed to describe and analyse motion.

GRAPHICAL ANALYSIS

The head-to-tail method used for adding and subtracting vectors in one dimension can also be used to add and subtract vectors in two dimensions. The difference is that the direction of the vectors must be indicated by a bearing or angle instead of a plus or minus sign. Compass bearings (such as south-west or S45°W), true bearings (such as 225°T) or simple angles are used in problems to indicate directions.

In the example in Figure 3.1.1, the vectors are drawn to a scale of 1 cm for every 5 N. A protractor is needed to draw the 45° angle and measure the angle of the resultant. This method can be used to add or subtract any number of vectors.
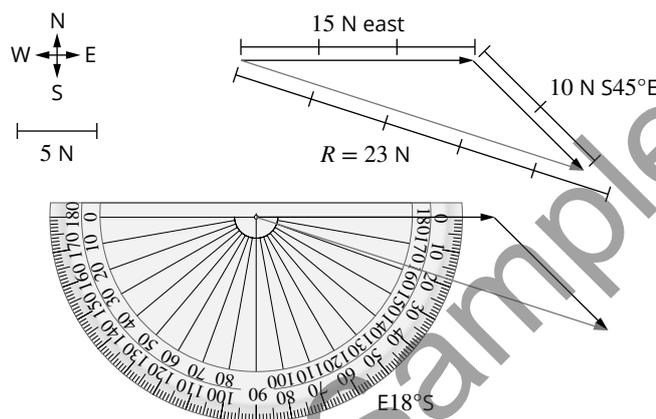

FIGURE 3.1.1 A graphical method can be used for adding vectors in two dimensions.

ALGEBRAIC ANALYSIS

Vectors at right angles to each other can be added and subtracted using Pythagoras' theorem and the trigonometric ratios of a right-angled triangle, as shown in Figure 3.1.2.

To calculate magnitude of the resultant vector, **Pythagoras' theorem** is used:

$$a^2 + b^2 = c^2$$

where:

c is the length of the hypotenuse

a and b are the lengths of the shorter sides.

To find the direction of the resultant vector, the appropriate trigonometric ratio is used:

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$
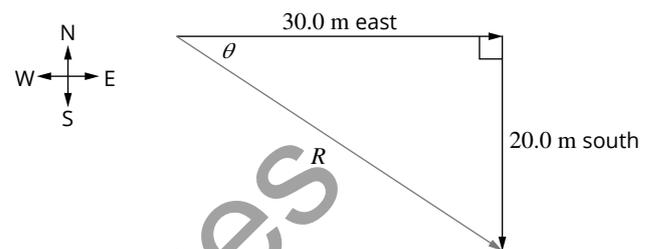
FIGURE 3.1.2 Two vectors at right angles can be added using the geometric method.

VECTOR COMPONENTS

Consider someone pulling a cart, as shown in Figure 3.1.3. The force exerted on the cart can be resolved (broken up) into two perpendicular components: a vertical component and a horizontal component.
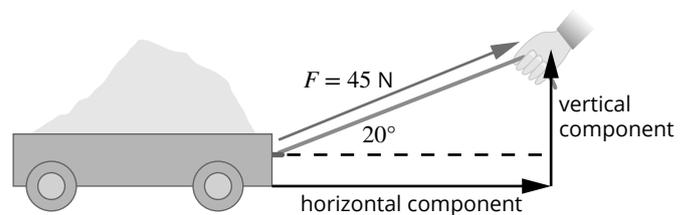

FIGURE 3.1.3 Vector components: pulling a cart

The perpendicular components are at right angles to each other and therefore make a right-angled triangle, with the original force vector forming the hypotenuse. This means that some properties of right-angled triangles can be used to help find the magnitudes of the components.

- Any component vector must be smaller in magnitude than the original vector, because the hypotenuse of a right-angled triangle is the longest side of the triangle.
- Graphically, a right-angled triangle vector diagram can be drawn with the original vector as the hypotenuse and the perpendicular components drawn from the tail of the original to the head of the original.
- Algebraically, the magnitudes of the perpendicular components can be found using the trigonometric ratios.

Resolving vectors into components is an important tool for analysing the effects of two or more forces acting on an object. For example, if an additional friction force is applied to the wheels of the cart in Figure 3.1.3, the force affects only the horizontal component. The new horizontal component can be easily calculated and then recombined with the unchanged vertical component to find the new resultant force vector.

Projectile motion

A **projectile** is any object that is thrown or projected into the air and has no power source driving it. If air resistance is ignored, the only force acting on the projectile is its **weight**, i.e. the force of gravity, F_g. This force is constant and always directed vertically downwards with an **acceleration** of $9.8\,\mathrm{m\,s^{-2}}$. Projectiles move in parabolic paths that can be analysed by considering the horizontal and vertical components of the motion (Figure 3.1.4). The two factors that affect a projectile's motion are the angle it is launched at and the initial **velocity**.

FIGURE 3.1.4 The motorcycle and rider are travelling in a parabolic path as they fly through the air.

PROJECTILES LAUNCHED HORIZONTALLY

Projectiles can be launched at any angle. The launch velocity needs to be resolved into vertical and horizontal components using trigonometry to complete most problems. For projectiles launched horizontally, the horizontal velocity is constant and equal to the

horizontal launch velocity; the initial vertical velocity is zero (but it will increase during the flight).

The parabolic path is traced by an object accelerating only in the vertical direction while moving at constant velocity in the horizontal direction. The following equations of motion for uniform acceleration must be used for the vertical component of the motion:

> ℹ️ $v = u + at$
>
> $s = ut + \dfrac{1}{2}at^2$
>
> $v^2 = u^2 + 2as$
>
> where:
>
> s is the **displacement** (m)
> u is the initial velocity ($\mathrm{m\,s^{-1}}$)
> v is the final velocity ($\mathrm{m\,s^{-1}}$)
> a is the acceleration ($\mathrm{m\,s^{-2}}$)
> t is the time (s).

The horizontal velocity of the projectile remains constant throughout its flight if air resistance is ignored. Therefore, the following equation for average velocity can be used for this component of the motion:

$$u_x = \frac{s_x}{t}$$

When solving a problem involving projectile motion, construct a diagram in which the direction is clearly specified (whether up or down is the positive or negative direction). Write out the information supplied for the horizontal (x-direction) and vertical (y-direction) components separately.

For example, a ball is hit horizontally from the top of a 10 m high cliff with a speed of $5.0\,\mathrm{m\,s^{-1}}$ (Figure 3.1.5). Use acceleration due to gravity of $9.8\,\mathrm{m\,s^{-2}}$ and ignore air resistance, to calculate the time it takes the ball to land and the distance it has travelled from the base of the cliff. To do this there are a few steps you should take.

First write out the variables that are known. You can then decide on the equations of motion that are best suited to finding the answers. Remember to keep your horizontal and vertical components separate.
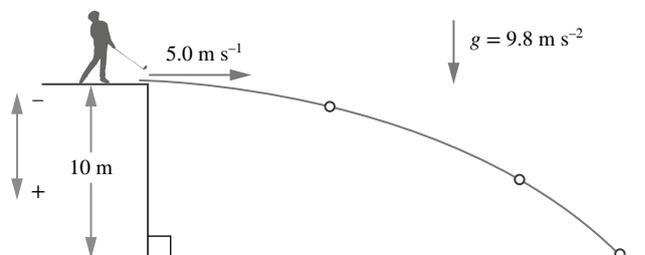

FIGURE 3.1.5 Parabolic motion with initial horizontal velocity only

Human cannonballs—a data analysis of projectile motion

In the days before WorkSafe, human 'cannonballs' used to be a fairly common feature of circuses and fairs. Not surprisingly, risk assessments, fair work practices and ever-increasing insurance premiums almost brought an end to them. More than 30 people died over the years since the first human cannonball, a 14-year-old girl, was launched at the Royal London Aquarium in 1877.

These days, a very few stunt performers maintain the tradition. The human cannonball is fired from a specially designed cannon, and lands on a horizontal net or inflated bag. The official world record for farthest distance stands at 59.43 m.

The data below was collected from the launch of a human cannonball of mass 80 kg blasted towards the east from a cannon and landing in a safety net. The height of the net was equal to the launch height. The angle of launch was 45° and the initial speed was 28 m s^{-1}. In your calculations, ignore air resistance unless otherwise instructed and use $g = 9.8$ m s^{-2}.

TABLE 1 Speed versus time after launch

Time after launch (s)	Speed (m s^{-1})
0	28.0
1.0	22.4
2.0	20.0
3.0	22.4
4.0	28.0

TABLE 2 Air resistance versus distance fallen

Air resistance (N)	Distance fallen (m)
0	0
400	50
600	100
700	150
780	200
830	250
850	300
870	350
890	400
900	450
900	500
900	550
900	600

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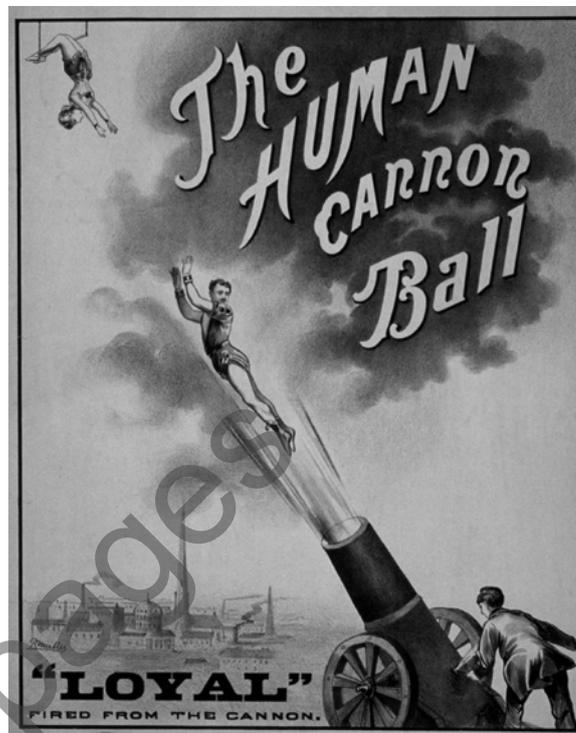
WORKSHEET 3.1.3

1 On the grid provided, plot a graph of vertical acceleration versus time for the human cannonball for the 4.0 s of motion. Use up as positive and ensure that your graph is appropriately labelled.
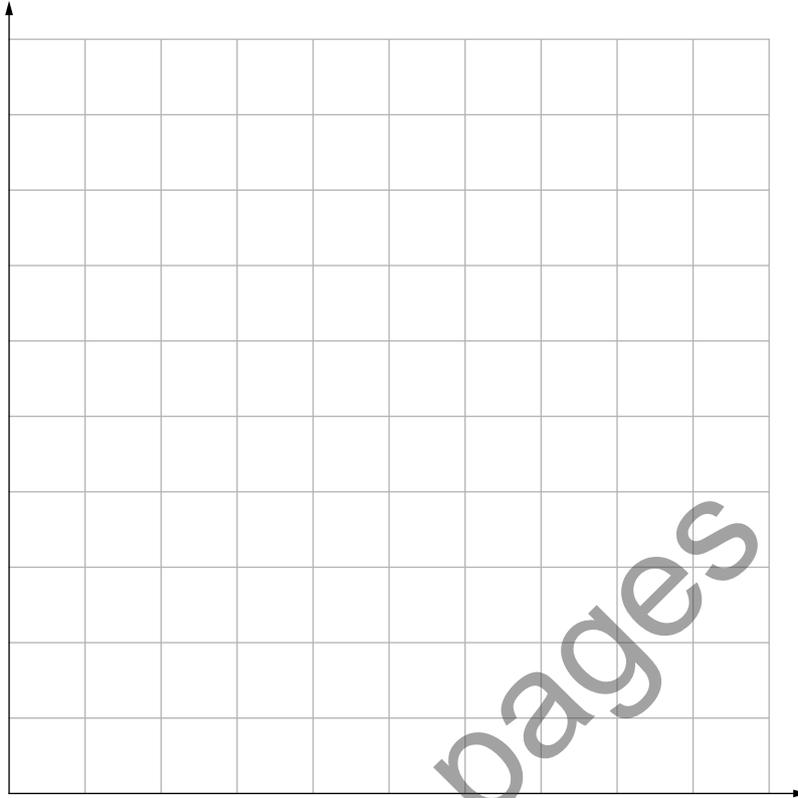
2 Use an energy approach for the following questions.

   a Determine the total mechanical energy of the human cannonball.

   _____

   _____

   _____

   b Determine the gain in gravitational potential energy of the human cannonball as they reach maximum height.

   _____

   _____

   _____

   _____

   c Determine the change in height that the human cannonball achieved from launch.

   _____

   _____

3 Determine the height of the cannonball above the initial launch height after:

   a 2.0 s

   _____

   _____

    b   3.5 s

4 Calculate the speed of the human cannonball after:

    a   2.0 s

    b   3.5 s

5 Determine the velocity and acceleration of the human cannonball at the maximum height achieved.

6 Discuss the forces that are acting on the human cannonball when passing through the highest point in the trajectory if air resistance is ignored. State the magnitude and direction of the net force acting at this point.

7 Discuss the forces that are acting on the human cannonball when passing through the highest point in the trajectory if air resistance is taken into account. Use diagrams where appropriate to assist in your explanation.

RATE MY LEARNING	• I get it. • I can apply/teach it.	• I get it. • I can show I get it.	• I almost get it. • I might need help.	• I get some of it. • I need help.	• I don't get it. • I need lots of help.

WORKSHEET 3.1.4

Dashing around the corner

In order for a body to travel in a circle or part of a circle, a centripetal force is required to be exerted on the object. This can be the pull of gravity for planets, the tension in a wire for a hammer thrower, or the reaction force of a surface in contact with the object. In the case of a car going around a corner, the centripetal force must come from the sideways component of reaction from the road surface or the friction of the car tyres against the road surface.

The diagram below depicts a car travelling away from the viewer and turning to the left.

1   Draw and label the force vectors acting on the car at this instant.

2   Consider a car negotiating a suburban roundabout. The path taken has a radius of curvature of 8.0 m and the coefficient of friction between the tyre and the road is 0.72. Calculate the maximum speed the car can travel (in km h^{-1}) and still remain on a circular path.

3   The coefficient of friction between the tyre of a Formula One racing car and the asphalt track is typically 1.50. Calculate the minimum radius of curvature that can be taken by a Formula One car travelling at a speed of 120 km h^{-1}.

4   Formula One cars are fitted with various aerofoils and body shaping to generate a down force. This helps control the car and keep it from lifting off the track. Determine how much down force (as a percentage of the car's weight) would be needed to negotiate the same corner at a 30% greater speed.

5 When a motorcycle goes around a corner, the rider needs to lean over into the corner. Draw and label the forces acting on the motorcycle and the rider on the diagram of a cornering motorcycle below. (Point C, 0.65 m from the contact point, is the centre of mass of the bike and rider.)
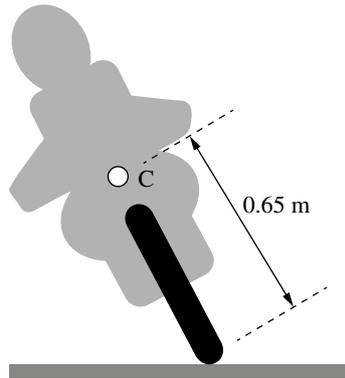
6 Determine the angle of lean of the motorcycle from the vertical as it takes the corner, assuming that the motorcycle's tyres can generate the same amount of friction with the road as the car's tyres.

_____

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7 Bicycles can travel faster around a bend on a 'banked' track as used in a velodrome. The ideal situation on a banked curve is for the bicycle to be perpendicular to the track surface. The Anna Meares Velodrome in Brisbane was the venue for the track cycling in the 2018 Commonwealth Games. The 250 m long timber track has a maximum angle of bank in the end curves of 43.9°, while the straights are angled at 12.5°.

Stephanie Morton, the winner of the women's individual sprint title, takes a path with a radius of curvature of 22 m at a point where the track is banked at the maximum angle. Calculate the greatest speed at which she can travel safely.

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Of course, there are many further points beyond this relatively simple scenario to consider when investigating how vehicles can be helped to negotiate corners.

• When a car first enters a bend and when it leaves the bend, it is usually travelling faster than when in the middle of the bend. This means the radius of curvature of the road needs to vary, and should be smallest at the apex of the corner.

• The radius of curvature of a velodrome track also changes as the banking angle varies from the straights to the ends.

• The greater the angle of bank, the harder a car is 'pushed' down onto the track. If pushed too far, the car body will touch the track and the tyres may lose adhesion.

• If a cyclist leans on a banked curve, they can generate more friction and take the curve even faster—provided they can pedal fast enough!

RATE MY LEARNING	• I get it. • I can apply/teach it.	• I get it. • I can show I get it.	• I almost get it. • I might need help.	• I get some of it. • I need help.	• I don't get it. • I need lots of help.

PRACTICAL ACTIVITY 3.1.4

Circular motion—centripetal force in a vertical plane

Suggested duration: 50 minutes

Research and planning

AIM

To investigate the non-uniform nature of the forces acting in vertical circular motion.

RATIONALE

In this activity the centripetal force experienced by an object travelling in a vertical plane will be investigated. This is an example of non-uniform circular motion. The techniques employed can also be used for uniform circular motion in a horizontal plane and to model orbital paths.

> ℹ️ As an option for more accurate timing, phones and tablets can be used to video the motion for video analysis. Use a bright-coloured mass against a plain background to allow for auto tracking by video analysis tools.

MATERIALS
- string
- small mass
- alligator clip
- stopwatch
- metre ruler

SAFETY

Always check around you and leave additional space before swinging objects around. Check that all knots are secure and the thread or line is new and without obvious imperfections before starting.

Students must wear safety glasses.

METHOD

1   Tie the string to the mass and measure the length of the string to the centre of the mass. Mark the string in 20 cm segments.

2   To reduce the friction between the string and your fingers, fasten the alligator clip at one of the 20 cm marks. This will give a better defined radius of the mass and reduces friction, allowing a more constant speed of rotation and improving accuracy.

3   Rotate the mass in a vertical circle. With practice, you can feel the change in tension in the string at the top of the circle compared with that at the bottom. Keep the rotation as even as possible.

4   Time the period of rotation for 20 revolutions and then find the average period. Repeat five times for each particular radius. You may need to use some additional paper if results are recorded for more than four radii.

5   Record your results in Table 1. Include estimates of the uncertainty in each measurement.

VARIABLES

List the variables in this experiment:

Independent: _____

Dependent: _____

Controlled: _____

....

PRACTICAL ACTIVITY 3.1.4

Analysing

TABLE 1 Radius and period

Radius, r (m) ±	Time for 20 revolutions (s) ±			Average time for 20 revolutions (s)	Period T for one revolution (s)
	Trial 1	Trial 2	Trial 3		
				±	±
				±	±
				±	±
				±	±

1. Plot a graph of period versus radius on the grid provided. Include appropriate error bars based on the calculated uncertainties in each measurement.
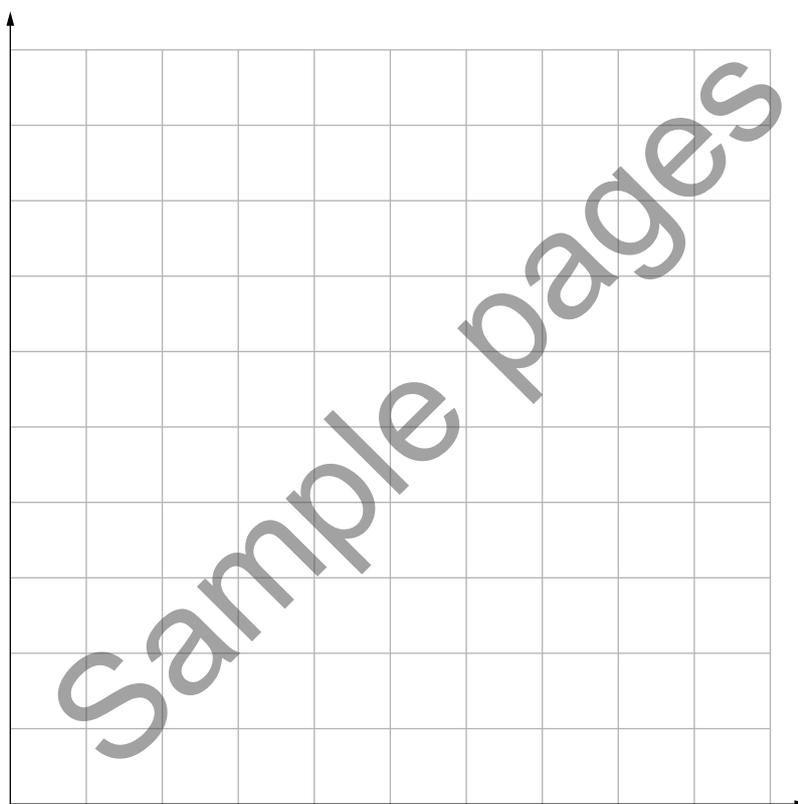
2. Explain why a graph of period versus radius is plotted.

_____

3. Calculate the gradient of the line. Use lines of best fit with minimum and maximum gradients to determine a measure of the uncertainty in your result.

_____

_____

4. Explain why vertical circular motion is described as non-uniform despite the period and the radius being constant through one or more revolutions.

_____

_____

PRACTICAL ACTIVITY 3.1.4

Interpreting and communicating

CONCLUSION
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1   Based on your graph, comment on the relationship between period and radius. State whether your results confirm what was expected from theory. Comment on any discrepancies.

_____

2   Identify the largest potential source of uncertainty in the experiment. Discuss how the experiment methodology endeavours to minimise the sources of uncertainty and suggest means by which they could be further reduced.

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RATE MY LEARNING	• I get it. • I can apply/teach it.	• I get it. • I can show I get it.	• I almost get it. • I might need help.	• I get some of it. • I need help.	• I don't get it. • I need lots of help.

Projectile motion—the effect of launch angle on range

Suggested duration: 40 minutes

Research and planning

AIM

To investigate the relationship between the launch angle of a projectile, its motion and the range of the projectile.

RATIONALE

- A projectile is any object that moves, without propulsion, in free flight. If air resistance is ignored, the only force acting on a projectile during its flight is that due to gravity. This force is constant and is always directed vertically downwards. It causes the projectile to follow a parabolic path.

- The motion of a projectile can be examined by looking at the horizontal and vertical components separately.

- Vertically, a projectile will move with an acceleration due to gravity ($9.8\,\text{m s}^{-2}$ downwards at Earth's surface).

- The horizontal component of velocity is uniform since, if air resistance is ignored, there are no forces acting in this direction.

- If a projectile is launched at an angle to the horizontal, trigonometry can be used to find the initial horizontal and vertical components of the velocity. The equations of motion can then be used to calculate the horizontal distance travelled by the projectile.

> **MATERIALS**
> - data-collection system
> - projectile launcher (commercial or improvised, e.g. poly tube)
> - projectile
> - photogate/s and (optional) time-of-flight pad or stopwatch
> - angle indicator
> - tabletop or bench
> - table clamp or burette stand and clamps
> - A4 paper
> - tape measure
> - sticky tape
> - carbon paper (optional)

SAFETY

- Always wear safety glasses when using any kind of projectile launcher. Never look down the barrel of a mechanical projectile launcher.

- At all times avoid pointing the projectile launcher towards any person.

METHOD

1   Start a new experiment on your data-collection system. Connect the photogates to your system following the manufacturer's instructions.

 Note: If photogates aren't available to you, a stopwatch can be used to find the flight time. In estimating the uncertainty in the measurement, be sure to allow for your reaction time when both starting and stopping the watch.

2   Select 'velocity between gates' if prompted by your data-collection system.

 Ensure the 'space between gates' parameter on your data-collection system is set to the measured space between your photogates.

3   Attach the projectile launcher to a table so that the projectiles travel across the longest part of the table. One suitable arrangement of launcher, photogates and table is shown in the set-up on the next page. Use the equipment available to you to arrange the launcher to 'fire' down the length of the table and through the photogates. Be careful to avoid firing the projectile at classmates!
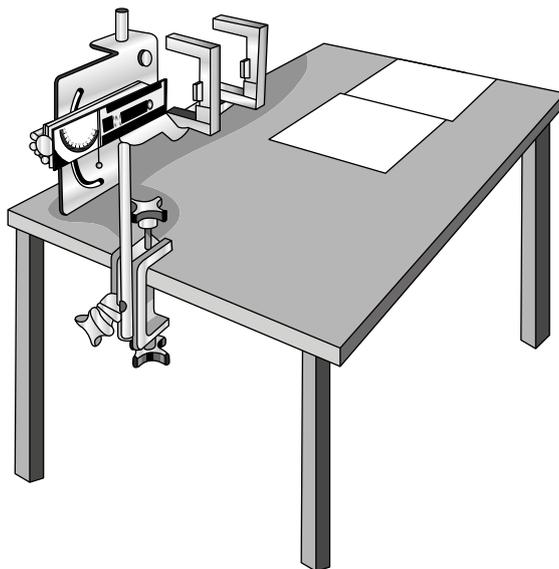
If a spring-loaded projectile launcher isn't available to you, a piece of curved 'poly pipe' supported by a retort stand can make a good alternative. Discuss, with the aid of diagrams, how you could do this.

_____

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_____

4   Place sheets of paper end-to-end in a line across the length of the table in front of the projectile launcher, and secure them in place with tape.

5   Measure the height from the point the ball is released to the tabletop, and record this value in the Analysing section.

6   Mount the photogates on the launcher. Be sure to mount the first named photogate in your data-analysis software closest to the launcher.

Part 1—Distance versus angle

7   Set the launcher in the horizontal position with a launch angle of 0°. Record an estimate of the uncertainty in the measurement of the angle in Table 1.

8   Load a projectile into the launcher, and ensure that the launcher is set to its maximum compression or distance setting.

9   Launch the projectile, and note the point of impact on the paper.

10 Lay a sheet of carbon paper on top of the white paper over the point of impact, carbon side down so that when a ball lands on it there will be a mark on the paper. Place a sheet of paper over the carbon paper to prevent damage to the carbon paper by the projectile.

   If carbon paper is not available, look for a small indentation on the paper where the ball hits or use tracing paper heavily shaded with lead pencil. Highlight the point with a pencil or marker when the projectile lands.

   Based on theory, predict the launch angle that you expect will yield the greatest horizontal range (distance).

_____

11 Start recording with the data-collection system (or your stopwatch) and launch the projectile.

Record the sampled 'velocity between gates' data point or time of flight, and enter the corresponding angle value in Table 1.

12 Move the carbon paper, and measure the distance from the base of the launcher to the mark. Write the angle next to the mark on the paper.

13 Use the angle indicator on the launcher to position the launcher at the next angle, 10°.

14 Repeat the data-collection steps, increasing the angle of inclination by 10° each time until you have recorded a data point every 10° from 0° to 80°.

15 Repeat the measurement for each angle an additional two times and find the average and uncertainty in the range.

16 Measure and enter the horizontal distance for each angle value into Table 1 in the Analysing section. Draw a graph of distance versus angle and a graph of velocity versus angle on the grids provided.

Part 2—Time of flight

17 If a time-of-flight pad is available, remove one photogate and attach the time-of-flight pad. Alternatively, a handheld stopwatch or other timing mechanism can be used. Position the time-of-flight pad over the landing point recorded for an angle of 0° and reset the launcher to an angle of 0°.

Based on your understanding of theory, predict the angle you think will give the greatest time of flight. Explain the reasons behind your prediction.

_____

18 Start a new data-collection session. Launch the ball from the launcher and using the time-of-flight pad, or a stopwatch, record the time the ball is in flight in Table 2.

19 Use the angle indicator on the launcher to position the launcher at the next angle, and repeat the data-collection steps until you have recorded a data point every 10° from 0° to 80°. Record the time of flight for each angle in Table 2.

20 Repeat the measurement for each angle an additional two times and find the average and uncertainty in the range.

21 Draw a graph of time of flight versus angle on the grid provided.

VARIABLES

List the variables in this experiment:

Independent: _____

Dependent: _____

Controlled: _____

Analysing

### RAW AND PROCESSED DATA

Ball release height: _____ (m)

TABLE 1 Projectile data for launch angle versus distance

Angle (°) ±	Velocity (m s^{-1})				Distance (m)			
	Trial 1	Trial 2	Trial 3	Average	Trial 1	Trial 2	Trial 3	Average
				±				±
				±				±
				±				±
				±				±
				±				±
				±				±
				±				±
				±				±
				±				±

TABLE 2 Projectile data for launch angle versus time of flight

Angle (°) ±	Time of flight (s)			
	Trial 1	Trial 2	Trial 3	Average
				±
				±
				±
				±
				±
				±
				±
				±
				±

Distance versus angle

Initial velocity versus angle
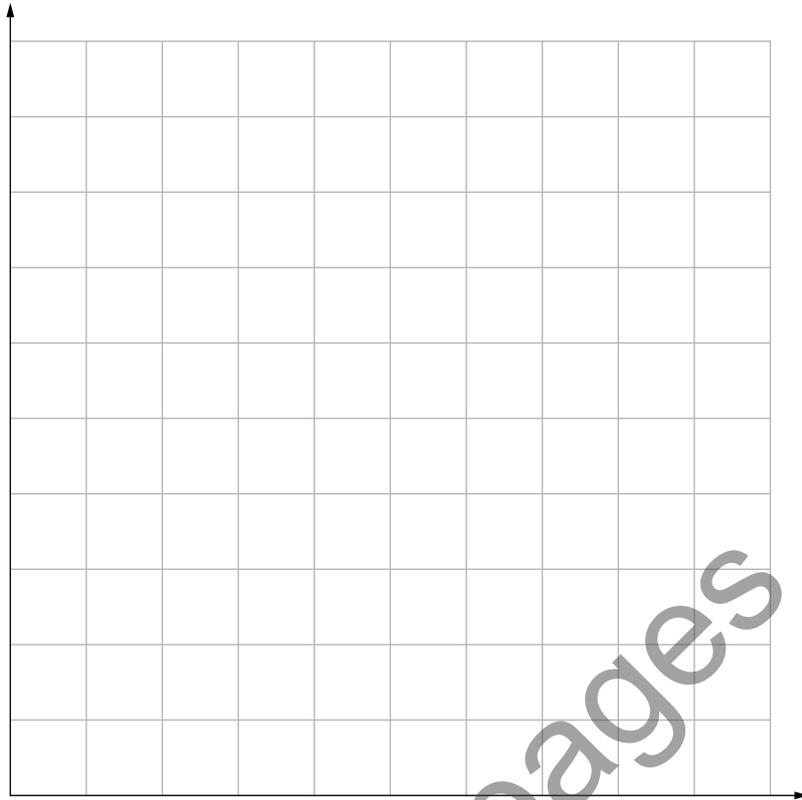
Time of flight versus angle
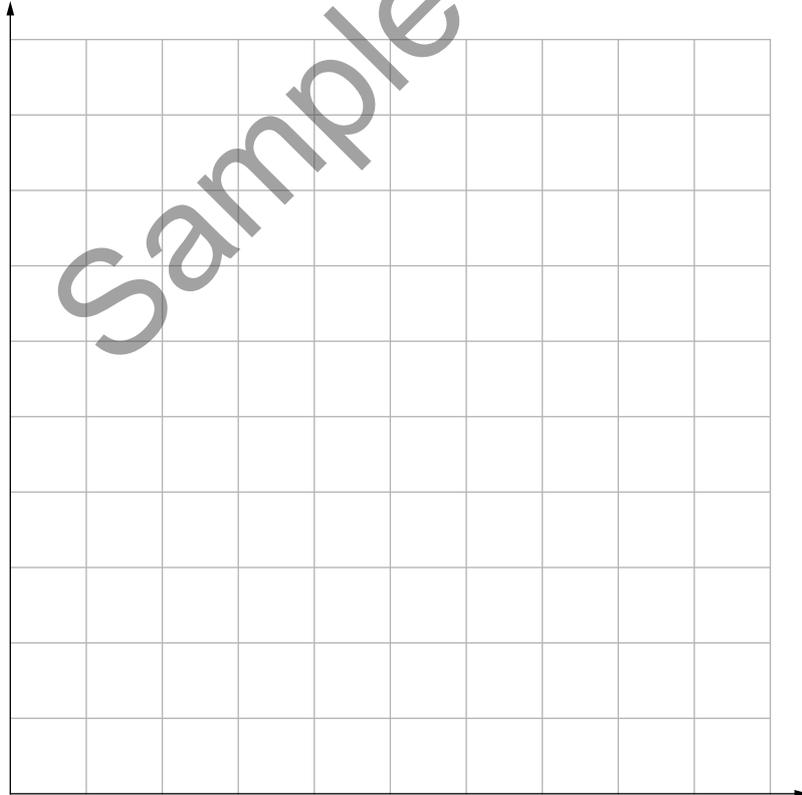
ANALYSIS

1   Choose one of the angles other than 0°, and draw to scale a vector diagram for your projectile at the launch position, showing both the horizontal and vertical component velocities. Show how you would determine the average initial velocity, and draw the resultant vector.

2   Calculate the horizontal and vertical components of velocity for each angle, and complete the columns in Table 3.

3   Use the horizontal distance and time of flight to calculate the average horizontal velocity for each angle, and fill in the corresponding column in Table 3. Explain why this is referred to as the average horizontal velocity.

4   Use the vertical velocity and the height of the launcher for each launch angle to calculate the theoretical time of flight of an object shot straight up. Record the results in Table 3.

TABLE 3 Projectile data calculations

Angle (°)	Horizontal velocity (m s^{-1})	Vertical velocity (m s^{-1})	Average horizontal velocity (m s^{-1})	Theoretical time of flight (s)

Interpreting and communicating

CONCLUSION

1   Discuss the degree to which the measured horizontal velocities from Table 1 compare with the calculated horizontal velocities from Table 3.

2 For any projectile launched horizontally, state what happens to the horizontal velocity as the projectile is in flight (a) in an ideal scenario and (b) in your experiment.

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_____

_____

_____

3 Based on your experimental results, state the launch angle that will yield the maximum horizontal range.

_____

_____

EVALUATION

4 Consider whether the reliability with which you could set and measure the angle of launch permitted reasonable conclusions to be drawn in this experiment. Discuss the effect of the uncertainty in your method on the reliability of your conclusion.

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Improvements and extensions

1 Identify the major sources of error in this experiment and the steps taken to minimise them.

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_____

_____

2 Consider whether there are launch angles that will yield the same horizontal range. Identify the angles (if any) and explain, using calculations where necessary, why they occur.

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RATE MY LEARNING	• I get it. • I can apply/teach it.	• I get it. • I can show I get it.	• I almost get it. • I might need help.	• I get some of it. • I need help.	• I don't get it. • I need lots of help.